golden pancake drama:

A linearly stable three-body orbit on an unequal-mass isola

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Abstract

We report high-precision numerical evidence for a collisionless planar Newtonian three-body orbit, named *golden pancake drama*, with masses $(1, 0.612356088827994835\ldots, 1)$ and zero angular momentum. An independent 160-bit Cartesian calculation supports full labeled inertial periodicity and planar linear stability. Targeted continuation gives a numerically closed mass isola in the Euler-half-twist symmetric sector, with middle mass approximately $0.468918 \le \mu \le 0.634604$. Its primitive shape word is `AABAbbaabaBB`; a second segment of the same continuation loop has threefold figure-eight topology. Near-collision calculations support binary collision transitions between these segments. No counterpart was identified in the audited reference catalogues. The results establish numerical family structure and finite-catalogue novelty evidence, not an interval existence proof, nonlinear stability theorem, or exhaustive publication-priority claim.

1 Model and initial conditions

Numerical catalogues have greatly expanded the known planar periodic solutions of the three-body problem [1, 2, 3, 4]. The present search targeted orbit families beyond repeated traversals and ordinary parameter samples of catalogued solutions. We consider three positive point masses in the plane with the unsoftened Newtonian force,

$$\ddot{\mathbf{q}}_i = \sum_{j\neq i} m_j \frac{\mathbf{q}_j - \mathbf{q}_i}{\|\mathbf{q}_j - \mathbf{q}_i\|^3}, \qquad G = 1, \qquad (m_1, m_2, m_3) = (1, \mu, 1). \tag{1}$$

The name *golden pancake drama* refers to the stable representative formerly identified in the numerical archive as `raw_iso_10_001128`. An Euler-phase representative has

$$\begin{aligned} &\mathbf{q}_1 = (-1, 0), && \mathbf{q}_2 = (0, 0), && \mathbf{q}_3 = (1, 0), \\ &\dot{\mathbf{q}}_1 = (u, v), && \dot{\mathbf{q}}_2 = -\frac{2}{\mu}(u, v), && \dot{\mathbf{q}}_3 = (u, v). \end{aligned} \tag{2}$$

These data enforce zero center of mass, linear momentum and angular momentum algebraically. Table 1 gives the orbit parameters. Its energy and scale-invariant period are

$$E = \left(1 + \frac{2}{\mu}\right)(u^2 + v^2) - 2\mu - \frac{1}{2} \simeq -1.54324896745656, \qquad T^* = T|E|^{3/2} \simeq 16.95847683810896. \tag{3}$$

Parameter	Value in the normalization of Eq. (2)
μ	`0.6123560888279948351624851400415065255699 67`
u	`0.151837333844166249473182946117144690707934`
v	`0.139577137807739642861113922044425748818806`
T	`8.845713110998389201297565455677696063171825`

Table 1: Initial-condition parameters, rounded for presentation. The companion machine-readable file retains the archived precision. T is the full labeled inertial period; rescaling to $E = -1$ gives period T^*.

2 Numerical periodicity and linear stability

The orbit was found by symmetry-constrained shooting initialized from an independently generated mixed Fourier loop. An isosceles exchange/time-reversal chart enforced $L = 0$; its state, middle mass and half-period were corrected jointly. An Euler phase was then verified at a quarter-period and transformed to Eq. (2).

High-order Taylor integration and Newton correction used 160-bit arithmetic. The result was independently checked in a 12-dimensional canonical Cartesian formulation, together with its variational equations. Full labeled closure was tested directly; closure in a rotating frame was insufficient. For the validated phase/scale representative, the normalized maximum position/velocity return error was 1.52×10^{-36}. Sampled energy and angular-momentum errors were approximately 1.01×10^{-40} and 1.90×10^{-42}, respectively. An independent DOP853 trajectory supplied a further consistency check.

After removing the neutral directions associated with the integrals and continuous symmetries, the four nontrivial Floquet multipliers were

$$\begin{aligned} \lambda_{1,2} &= -0.4723571663700045541 \ \pm\ 0.8814072312948764540\, i, \\ \lambda_{3,4} &= 0.0121176023770802782 \ \pm\ 0.9999265791610056899\, i. \end{aligned} \tag{4}$$

Their moduli differed from unity by less than 8×10^{-36} in the high-precision computation. We therefore classify the orbit as numerically *planar-linearly stable*. This statement does not establish nonlinear or KAM stability, nor is the orbit's existence certified by interval arithmetic.

3 Topology and the isolated mass branch

Native 192-bit event detection resolved 18 syzygies, with reduced cyclic itinerary `121231323121231323`. Using positive collision-puncture meridians $a, b, c = (ab)^{-1}$, and capitals for inverses, the canonical free-group word is

$$W = \mathtt{AABAbbaabaBB}. \tag{5}$$

Published generator conventions were normalized before matching. Equivalence tests included cyclic base-point shifts, body permutations, spatial reflection and time reversal. This nonempty word is not a proper power; conditional on the validated loop, the reported orbit is therefore not a multiple traversal of a shorter labeled orbit.

Pseudo-arclength continuation, with local correction and native topology checks, was performed in both mass directions. High precision was used near close encounters. The two directions join at $\mu = 0.55$, with a maximum parameter discrepancy of 1.90×10^{-29} and opposite tangents. Independent Cartesian checks verify this shared orbit and the lower mass fold. The observed rank-three continuation Jacobian and the joined paths support a closed isola in the Euler-half-twist, $L = 0$, $(1, \mu, 1)$ sector (Fig. 1). Its refined mass extrema are

$$\mu_{\min} \simeq 0.4689179813307963, \qquad \mu_{\max} \simeq 0.6346042508062549. \tag{6}$$

Thus this particular symmetric branch never reaches three equal masses. The result is not a global exclusion of paths through symmetry-breaking bifurcations or other mass distributions.

A second segment has word $(ABab)^3$, the topology of a threefold figure-eight satellite [6]. It is not merely a triple repetition: return tests including $T/3$ found no shorter labeled return at the shared landmark or lower fold. The topology changes near $\mu \simeq 0.50655$ and 0.62944. Opposite signs of close-pair relative angular momentum, pair separations down to about 1.9×10^{-7} and third-body distances exceeding 1.1 support binary-collision transitions. These are numerical collision-limit estimates, not certified collision solutions; exact collisions are excluded from the collisionless orbit catalogue. The cube topology alone does not identify descent from a specific published figure-eight orbit.

4 Novelty, scope and reproducibility

No matching stable-point word or published numerical counterpart was found in the audited sources. The comparison covered the 695/1349 catalogues [1, 2], the 971 stable and 12,431 Euler itineraries [4, 3], Rose's 221 collisionless entries [5], known figure-eight and BHH root classes, and the earlier project results. Of the 421,562-row Euler source, 11,633 overlapping initial-state tuples agreed with the published shorter

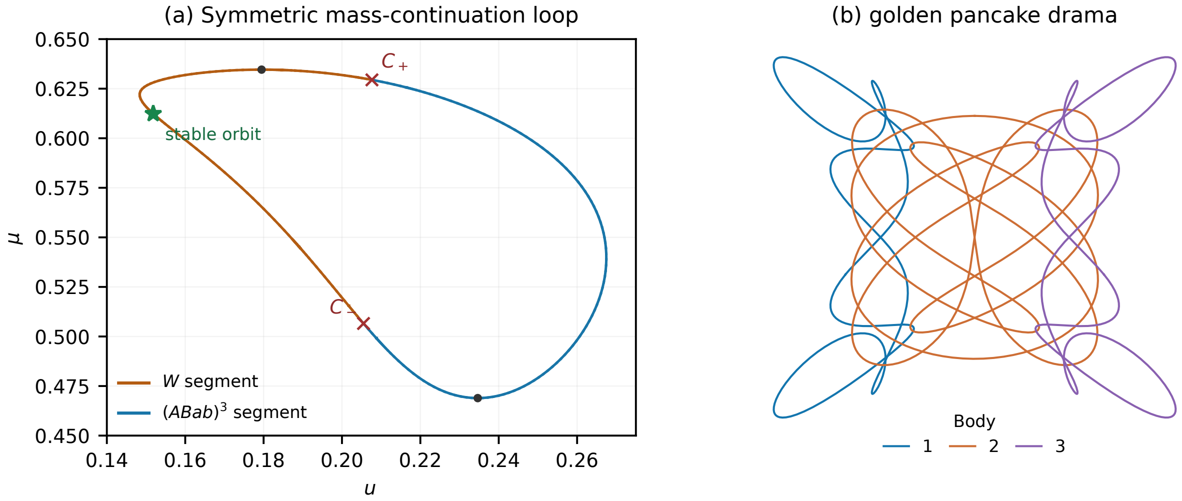

Figure 1: Left: the numerical mass-continuation loop, in the Euler normalization of Eq. (2). The marked point is *golden pancake drama*; orange denotes its primitive topology and blue denotes $(ABab)^3$. Crosses mark estimated collision limits. Only the marked orbit's linear stability is asserted. Right: its trajectories on fixed inertial axes, rescaled to $E = -1$.

catalogue at their common printed precision; all 272 additional rows with $T^* < 70$ passed targeted 160-bit event checks without a match to W or $(ABab)^3$. The higher-period topology coverage remains partial. The 821-satellite and 462-choreography tables contain no power-three examples [6, 7]; absence from those tables is not, by itself, proof of novelty.

The supported finding is an isolated unequal-mass family not represented in the audited sources, with a linearly stable segment of distinct primitive topology. Worldwide publication priority and complete global ancestry remain unproved. An earlier mass-stepping attempt changed word at $\mu \simeq 0.660356$; its endpoint near 0.716137 is explicitly rejected as ancestry evidence.

Companion files provide the exact initial conditions, figure data, validation summary and editable source. The project archive retains the numerical continuation paths, event records and proof hashes. No additional orbit search was performed for this report.

References

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